Theoretical Computer Science 4 (1977) 235.


NOTE

# A NOTE ON SCHNORR'S SEPARATEDNESS

Daniel J. LEHMANN

*Department of Computer Science, University of Warwick, Coventry CV4 7AL, England*

**Abstract.** It is shown that the two concepts of separatedness proposed by Schnorr are related: separatedness implies 1-separatedness. Schnorr's results on 1-separatedness then imply his results on separatedness.

In [1] Schnorr proposed the following two definitions (the notations are those of [1]).

**Definition 3.1.** Let $f \in \Omega$, then $B \subset \mathrm{mon}(f)$ is called *separated* iff $\forall r \in \mathrm{mon}(f) : \forall s, t \in B : r \geqslant s \cdot t \implies [r = s \text{ or } r = t]$.

**Definition 5.1.** A subset $B \subset \mathrm{mon}(f)$ is called 1-*separated* if (S1), (S2) hold.

(S1) $\quad \forall r \in \mathrm{mon}(f) : \forall s, t \in B : r \geqslant s \cdot t \implies [r \leqslant s \quad \text{or} \quad r \leqslant t]$.

(S2) $\quad \forall r \in \mathrm{mon}(f) : \forall s \in B : r \geqslant s \implies r = s$.

We claim:

**Theorem 1.** (S1) $\implies$ (S2).

**Proof.** Suppose $r \in \mathrm{mon}(f)$, $s \in B$ and $r \geqslant s$ then $r \geqslant s \cdot s$ and by (S1) $r \leqslant s$. But $r \geqslant s$ and $r \leqslant s$ imply $r = s$. As a consequence, the condition (S2) can be dropped in the definition of 1-separatedness. □

**Theorem 2.** *If a subset $B \subset \mathrm{mon}(f)$ is separated then it is* 1-*separated.*

**Proof.** Obvious. □

**Theorem 3.** *Let $f \in \Omega_{\cdot}$, $\bar{\#}_1(f) \geqslant \bar{\#}(f)$ and $\#_1(f) \geqslant \#(f)$.*

It follows that Schnorr's Theorem 5.2 implies his Theorem 3.2. Lemmas 3.4, 3.5 and 3.6 can be dispensed with.

## Reference

[1] C.P. Schnorr, A lower bound on the number of additions in monotone computations, *Theoretical Computer Sci.* **2** (1976) 305–315.